

MATHEMATICAL ANALYSIS 1
SYLLABUS & IMPORTANT THEOREMS

- Elements of logic
 - Proof by induction
 - Proof by contradiction
- Functions
 - Injectivity, surjectivity, invertibility
 - Monotonicity (also of sequences)
 - The elementary functions
- Sequences
 - Subsequences, The **Bolzano-Weierstrass Theorem**
- Complex numbers
- Limits
 - The number e
 - Asymptotes
 - The $\varepsilon - \delta$ formalism
 - Continuity and types of discontinuities
 - Left and right limits
 - Algebra of limits
 - Indeterminate (“meaningless”) limits
 - Comparison theorems
- Asymptotic behavior of functions
 - Landau symbols
 - Order of a function, principal part
- Continuous functions on intervals
 - **Bolzano’s Theorem** (existence of zeroes)
 - **Intermediate Value Theorem**
 - **Weierstrass’ Theorem**
 - Uniform continuity and Lipschitz
 - **Heine-Cantor Theorem**
 - Invertibility
- The derivative
 - Definition and rule
 - **Theorems of Rolle, Lagrange, Cauchy**
 - Higher-order derivatives
 - Convexity and inflection points
 - Qualitative study of a function
 - **De l’Hôpital’s Theorem**
- Taylor and Maclaurin expansions
 - Expansions of the elementary functions
 - **Peano and Lagrange remainders**
 - Local behavior of a function via its expansion
- The integral
 - Antiderivatives and indefinite integrals
 - Rules, **integration by parts, integration by substitution**
 - Definite integrals, **Cauchy integral, Riemann integral**
 - Integral mean value
 - **Fundamental Theorem of Integral Calculus**

- Integrals with functional limits
- Improper integrals and series
 - Improper integrals of type I
 - Improper integrals of type II
 - Improper integrals convergence tests
 - Series, geometric series
 - Series convergence tests