MATHEMATICAL ANALYSIS 1
HOMEWORK 2

(1) Prove Newton's binomial formula. *Hint: prove by induction.* In your proof you may also use the formula:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

(2) Let $f : X \to Y$ be a function between two sets X and Y.
- (a) Prove that $A \subseteq f^{-1}(f(A))$ for any $A \subseteq X$.
- (b) Give an example of when $A \neq f^{-1}(f(A))$.

(3) Describe the following subsets of $\mathbb{R}$, specify what are their infimum and supremum, and determine whether their minimum and maximum are attained (explain your answers).
- (a) $A = \{x \in \mathbb{R} \mid (x+1)(x-1)(x-5) < 0\} \cap \{x \in \mathbb{R} \mid \frac{3x+1}{x-2} \geq 0\}$
- (b) $B = \{x \in \mathbb{R} \mid x - 4 \geq \sqrt{x^2 - 6x + 5}\} \cup \{x \in \mathbb{R} \mid x + 2 > \sqrt{x-1}\}$
- (c) $C = \{x \in \mathbb{R} \mid x = \frac{1}{n-2},\ n = 3, 4, 5, \dots\}$

(4) Describe and sketch the following subsets of $\mathbb{R}^2$:
- (a) $A = \{(x,y) \in \mathbb{R}^2 \mid xy \geq 0\}$
- (b) $B = \{(x,y) \in \mathbb{R}^2 \mid 1 + xy > 0\}$
- (c) $C = \{(x,y) \in \mathbb{R}^2 \mid x^2 - y^2 > 0\}$
- (d) $D = \{(x,y) \in \mathbb{R}^2 \mid x - y \neq 0\}$
- (e) $E = \{(x,y) \in \mathbb{R}^2 \mid |x - y| < 2\}$
- (f) $F = \{(x,y) \in \mathbb{R}^2 \mid |x - y| < -2\}$

(5) Describe the following sets: (explain your answers)
- (a) $\mathrm{dom}\,(f)$ where $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = \frac{1}{\sin x}$.
- (b) $\mathrm{im}\,(f)$ where $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = \frac{1}{\sin x}$.
- (c) $\mathrm{dom}\,(f)$ where $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = \frac{1}{2+\sin x}$.
- (d) $\mathrm{dom}(f)$ where $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = 10^x$.
- (e) $\mathrm{im}(f)$ where $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = 10^x$.
- (f) $\mathrm{dom}(f)$ where $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = \log_{10}(x)$.
- (g) $\mathrm{im}(f)$ where $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = \log_{10}(x)$.

(6) For the following functions f and subsets $B \subseteq \mathbb{R}$, describe $f^{-1}(B)$.
- (a) $f : \{3, 4, 5, \dots\} \to \mathbb{R}$ defined by $f(n) = \frac{1}{n-2}$, and $B = (0, 1)$.
- (b) In the previous question, what if $B = [0, 1]$?
- (c) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^4$, $B = [1, 16)$.
- (d) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 + 1$, $B = (-26, -7]$.
- (e) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \cos x$, $B = \{0\}$.

(7) Compute $\frac{100!}{98!}$. Explain your answer.