MATHEMATICAL ANALYSIS 1
HOMEWORK 1

(1) Prove that there are infinitely many prime numbers (Euclid's Theorem).
(2) Prove that the solution of the equation $x^2 = 7$ is irrational.
(3) Prove that for any set X and for any two subsets $A, B \subset X$, we have $(A \cup B)^C = A^C \cap B^C$.
(4) Solve the inequality: $\sqrt{|x^2 - 4|} - x \geq 0$.
(5) Let $P \subseteq \mathbb{R}$ be the set of prime numbers.
   (a) Is P bounded from above? from below? what is its infimum? supremum? do either the supremum or infimum belong to P?
   (b) Let $Q = \{x \in \mathbb{R} \mid x^{-1} \in P\}$. Is Q bounded from above? from below? what is its infimum? supremum? do either the supremum or infimum belong to Q?
(6) Consider the following subset of $\mathbb{R}$:

$$A = \left\{ x \in \mathbb{R} \mid 0 \leq x < 1 \quad \text{or} \quad x = \frac{2n-3}{n-1},\ n \in \mathbb{N} \setminus \{0, 1\} \right\}.$$

Is A bounded from above? from below? what is its infimum? supremum? do either the supremum or infimum belong to A?