1.6 Types of Second Order Equations

We have already seen the three main types of 2ND order PDEs: wave, diffusion (heat) and for steady-states (i.e. solutions that don't change with time) both of these become the Laplace equation.

These represent three important types:

HYPERBOLIC: $u_{tt} = u_{xx}$ + lower order terms*

↓ ↓

Compare to $t^2 = x^2 + 1$ ← eq. for a hyperbola

PARABOLIC: $u_t = u_{xx}$ + lower order terms*

↓ ↓

Compare to $t = x^2 + 1$ ← eq. for a parabola

ELLIPTIC: $0 = u_{xx} + u_{yy}$ + lower order terms*

↓ ↓

Compare to $1 = x^2 + y^2$ ← eq. for an ellipse

* lower order terms = u_x, u, constants....

Many 2ND order equations can be transformed into one of these forms (up to some constants, perhaps, and some lower order terms).

Moreover, in all of these, the right hand side can be written as Δu in higher dimensions.