1.4 Initial and boundary conditions

In the previous example we saw that the PDE $u_x + \cos x \, u_y = 0$ has $u(x,y) = f(y - \sin x)$ as a solution, where f can be any function $\Rightarrow \infty$ of solutions. To identify a particular solution we had to specify an auxiliary condition.

This is true in general.

Types of boundary conditions:

(D) Dirichlet boundary conditions:
specifying the value of u somewhere (typically on the boundary)

(N) Neumann boundary conditions:
specifying $\frac{\partial u}{\partial n}$ on the boundary

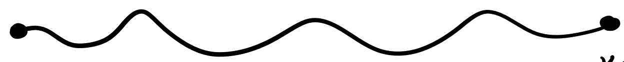
(R) Robin boundary conditions:
specifying $\frac{\partial u}{\partial n} + au$ on the boundary

Whenever any of these conditions is simply 0, we say that the condition is homogeneous.

For time dependent problems we also need to provide initial conditions about the state of the system at some time t_0.

Example: Vibrating string.

We need to specify boundary conditions and initial conditions.



So if the string is fixed at both ends we have:

$$\begin{cases} \partial_{tt} u - c^2 \partial_{xx} u = 0 & 0<x<\ell \\ u(0,t) = u(\ell,t) = 0 & t \geq 0 \\ u(x,0) = \phi(x) & 0<x<\ell \\ u_t(x,0) = \psi(x) & 0<x<\ell \end{cases}$$

Example: (in the homework)

$$\begin{cases} \partial_t u - \partial_{xx} u = f(x) = \begin{cases} 0 & 0<x<\ell/2 \\ H>0 & \ell/2<x<\ell \end{cases} \\ u(0,t) = u(\ell,t) = 0 & t \geq 0 \end{cases}$$

What is the steady state? As $t \to \infty$, a steady state will satisfy $\partial_t u = 0$, so that we're left with

$$-\partial_{xx} v = f(x) \quad \text{and} \quad v(0) = v(\ell) = 0$$

where we call the asymptotic state v instead of u.