Recent Progress in Collisionless Models: Abstracts

Claude Bardos (Paris Diderot)

About the Maxwell-Boltzmann relation for fluids and plasmas

This is a preliminary report on a joint work with François Golse, Claudia Negulescu and Remi Sentis. The standard description of the evolution of a plasma at the kinetic level involves the coupling of two Vlasov equations one for electrons and one for ions. The Maxwell-Boltzmann relation is used to replace the electron equation by a formula of the following type for the potential:

$$-\Delta\Phi + e^{\lambda\Phi} = \int f_{ions}(x, v, t)dv$$

The way to justify the formula consists in assuming that the electrons have reached some type of thermodynamically equilibrium. A formal proof can be obtained by taking in account the relaxation to a Maxwellian due collision term $\sigma_{electrons}C(f_{electron})$ (Boltzmann or Fokker Planck kernel) and looking for times large enough with respect to σ. Then at this formal level things are much easier than for the Landau damping. On the other hand these non linear effect make the complete proof more difficult and they may be only available for small fluctuations.

Julien Barré (Nice)

Perturbation of non homogeneous stationary states of Vlasov equation

Take a non homogeneous stationary state of a Vlasov equation, which is weakly unstable, and perturb it. What is the fate of this perturbation? After presenting potential applications, I will address this question using heuristic computations and numerical simulations. This is a joint work with David Métivier (University of Nice), and Yoshiyuki Yamaguchi (University of Kyoto).

Jacob Bedrossian (Maryland)

Landau damping in Gevrey regularity for Vlasov-Poisson and connections with hydrodynamic stability

We prove nonlinear Landau damping on the d-dimensional torus in Gevrey-$1/s$ regularity ($s > 1/3$), the “critical” regularity predicted by formal weakly nonlinear analysis by Mouhot and Villani. Our proof combines ideas from the original proof of Landau damping and the proof of stability of the Couette flow in the 2D Euler equations. In addition to obtaining the predicted regularity classes, our proof is simpler and more accessible than the original proof, replacing the Newton iteration with a priori energy estimates obtained with paradifferential calculus and a more optimal use of time-varying regularity. Differences and similarities with the recent results on stability and instability in the 2D and 3D Couette flow in the Navier-Stokes equations will be discussed if time permits. Joint work Nader Masmoudi and Clément Mouhot.

Nikolaos Bournaveas (Edinburgh)

Global existence and blow up for some kinetic and hyperbolic models of chemotaxis

We’ll discuss existence vs blow-up results for the Othmer-Dunbar-Alt kinetic model of chemotaxis as well as for a hyperbolic model.

Simone Calogero (Chalmers)

Relativistic diffusion

Recent progress and open problems on the relativistic theory of diffusion will be reviewed. The final goal is to introduce the Einstein-Vlasov-Fokker-Planck system with cosmological scalar field, a new model for the dynamics of kinetic particles undergoing diffusion in general relativity.

Martin Campos Pinto (Paris Pierre et Marie Curie)

On structure-preserving DG-PIC schemes for the Vlasov-Maxwell system

In this talk we describe a unified approach to design conforming and non-conforming Galerkin schemes with long-time stability properties for the Vlasov-Maxwell system.

For the pure Maxwell system first we identify a compatibility relation that takes the form of a commuting diagram that should be satisfied by the discrete curl operators, and for the coupled problem with approximate sources we show that some structure relations involving an exact sequence of discrete operators allow to characterize a proper continuity equation that should be satisfied by a coupled scheme.

The main novelty of our approach is that this framework is used to design new non-conforming schemes (such as DG-PIC with new current deposition schemes) that are stable over large time ranges.

Silvia Caprino (Roma Tor Vergata)

On a Vlasov-Poisson plasma with infinite charge and velocities

I will speak about the well posedness of the Vlasov-Poisson equation, when removing some classical assumptions on the initial data, as to belong to L^1 in space and to have compact support in the velocities.

Yingda Cheng (Michigan State)

Energy-conserving discontinuous Galerkin schemes for the Vlasov-Maxwell system

In this talk, we present the discontinuous Galerkin (DG) methods to solve the Vlasov-Maxwell system. The scheme employs DG discretizations for both the Vlasov and the Maxwell's equations, resulting in a consistent description of the probability density function and electromagnetic fields. We prove that using this description the total particle numbers are conserved, and the total energy could be preserved upon a suitable choice of numerical flux for the Maxwell's equations and the underlying polynomial spaces on the semi-discrete level, if boundary effects can be neglected. We further established error estimates based on several flux choices. We test the scheme on the Weibel instability and verify the order and conservation of the method.

Bruno Després (Paris Pierre et Marie Curie)

Advances in the modeling of kinetic sheath in plasma

Sheath are particular stationary physical solutions in a plasma near a metallic wall, where a boundary layer is not locally neutral. We will show a natural minimization formulation for kinetic Vlasov equations (for $\mathsf{i}+$ and $\mathsf{e}-$) plus a non linear Poisson equation. It yields the existence of sheaths. The mathematical stability will be discussed, plus some extention to more challenging problems in plasma. This work has been done at LJLL-UPMC with Mehdi Badsi (PHD) and Martin Campos Pinto (CNRS).

Irene Gamba (Austin)

On computational issues of Vlasov-Maxwell and Vlasov-Poisson-Landau

We discuss a hybrid numerical approach to Vlasov-Poisson-Landau for very mild collisional plasma systems. We propose a time splitting computational scheme that allows for a conservative approach for the whole system, where the Landau operator is computed by an spectral based solver, and both, the Vlasov-Poisson and Vlasov-Maxwell systems, are computed by a conservative Discontinuous Galerkin approach. The scheme, numerical analysis and simulations issues will be discussed. This is work in collaboration with Chenglong Zhang, as well as Yingda Cheng, Fengyan Li and Phil Morrison.

Igor Gapyak (Kyiv)

On the rigorous derivation of the Enskog kinetic equation

In this talk we consider a new approach to the rigorous derivation of the Enskog kinetic equation from hard sphere dynamics. We establish that for initial states which are specified in terms of the one-particle distribution function the evolution of all possible states described by the Cauchy problem of the BBGKY hierarchy for hard spheres can be described in the framework of the Cauchy problem of the generalized Enskog kinetic equation and by a sequence of explicitly defined functionals of a solution of such kinetic equation. To prove this result we develop the method of kinetic cluster expansions of cumulants of groups of operators of the Liouville equations which are determined the every term of a solution expansion of the BBGKY hierarchy. For the initial value problem of the generalized Enskog equation the existence theorem is proved in the space of integrable functions. We consider also the Markovian approximation of the collision integral of the generalized Enskog equation and establish its links with the collision integrals of the Enskog-type kinetic equations.

François Golse (École polytechnique)

On the mean-field and classical limits for the N-body Schrödinger equation

This talk proposes a quantitative convergence estimate for the mean-field limit of the N-body Schrödinger equation that is uniform in the classical limit. It is based on a new variant of the Dobrushin approach for the mean field limit in classical mechanics, which avoids the use of particle trajectories and empirical measures, and has a very natural quantum analogue. (Work in collaboration with C. Mouhot and T. Paul).

Yan Guo (Brown)

Derivation of steady Navier-Stokes equations from the Boltzmann theory

With new uniform L^3 estimates in bounded domains, we show that the steady Boltzmann solution converges to the steady Navier-Stokes solution in the diffusive limit.

Susana Gutierrez (Birmingham)

Strichartz estimates for the kinetic transport equation

We discuss general mixed-norm Strichartz estimates for the linear homogeneous kinetic transport equation. In particular, and in contrast with such estimates for the time-dependent free Schroedinger equation, we show that the conjectured endpoint inequality is false in all dimensions.

Daniel Han-Kwan (École polytechnique)

The quasineutral limit of the Vlasov-Poisson system

We will review some recent progress in the study of the quasineutral limit (i.e. the small Debye length regime) of the Vlasov-Poisson system.

Thomas Holding (Cambridge)

Instability of non-monotone equilibria of the relativistic Vlasov-Maxwell system on unbounded domains

In a series of papers in 2007-8, Lin and Strauss established a sharp linear stability criterion for equilibria of the Vlasov-Maxwell system. The existence of a growing mode solution of the linearised system was shown to be equivalent to the existence of negative eigenvalue(s) of a certain Schrödinger operator acting on the spatial variable alone (and not the full phase space). These results, however, applied only to *monotone* equilibria, and made heavy use of this assumption.

In this talk I will present sufficient conditions for *instability* that do not require monotonicity or the boundedness of the domain. The problem reduces to studying how the spectra of a family of unbounded self-adjoint operators depends upon a parameter, and a key step is the construction of explicit finite dimensional approximations to this family whose spectra converge in an appropriate sense. This is joint work with Jonathan Ben-Artzi.

Slim Ibrahim (University of Victoria)

The Vlasov-Poisson system for stellar dynamics in spaces of constant curvature

We derive the Vlasov-Poisson equations on symplectic manifolds, and we analyze the system when the motion is restricted on great circles.

Mohammed Lemou (Rennes)

On quantitative rearrangement inequalities and their applications to Vlasov-Poisson, HMF and 2D-Euler systems

In this talk, we will first present a new functional inequality (of Hardy-Littlewood type) for generalized rearrangement of functions, discuss its motivations and give the main lines of the proof. We then show how this inequality provides quantitative stability results of steady states to evolution systems that essentially preserve the rearrangements and some suitable energy functional. Two main examples of such evolution system are the gravitational Vlasov-Poisson (VP) and the 2D-Euler systems. The known stability results for these models are essentially based on compactness arguments which by construction provide no quantitative control of the perturbation. After giving a brief review of these known results, we will present the main application of this work by making completely quantitative these stability results: the perturbations of a large class of steady states for these models are shown to be quantitatively controlled in strong norms by perserved quantities, thus avoiding the use of compactness arguments. We then extend this strategy to the so-called HMF (Hamiltonian Mean Field) system for which physical steady states (with finite mass and energy) may have unbounded supports.

Zhiwu Lin (Georgia Tech)

Instability index, exponential trichotomy and invariant manifolds for Hamiltonian PDEs

Consider a general linear Hamiltonian system $u_t = JLu$ in a Hilbert space X, called the energy space. We assume that L induces a bounded and symmetric bi-linear form $< L\cdot, \cdot >$ on X, and the energy functional $< Lu, u >$ has only finitely many negative dimensions $n(L)$. There is no restriction on the anti-selfadjoint operator J, which can be unbounded and even with an infinite dimensional kernel space. Our first result is an index theorem on the linear instability of the evolution group e^{tJL}. More specifically, we get some relationship between $n(L)$ and the dimensions of generalized eigenspaces of eigenvalues of JL, some of which may be embedded in the continuous spectrum. Our second result is the linear exponential trichotomy of the evolution group e^{tJL}. In particular, we prove the nonexistence of exponential growth in the finite co-dimensional center subspace and the optimal bounds on the algebraic growth rate there. This is applied to construct the local invariant manifolds for nonlinear Hamiltonian PDEs near the orbit of a coherent state (standing wave, steady state, traveling waves etc.). For some cases, we can prove orbital stability and local uniqueness of center manifolds. We will discuss applications to examples including dispersive long wave models such as BBM and KDV, 2D Euler equation for ideal fluids, and Vlasov-Maxwell systems for collisionless plasmas. This is a joint work with Chongchun Zeng.

Jonathan Luk (Cambridge)

Global existence and blow up for some kinetic and hyperbolic models of chemotaxis

We consider the relativistic Vlasov-Maxwell system with initial data of unrestricted size and discuss the application of Strichartz estimates and moment bounds to this system. In two dimensions and two-and-one-half dimensions, we extend the global regularity results of Glassey-Schaeffer without assuming that the initial data have compact momentum support. In the three dimensional case, we apply these techniques to obtain a new continuation criterion which in particular guarantees that a regular solution can be extended as long as the kinetic energy density is in L^p for some $p > 2$. This is a joint work with Robert Strain.

Evelyne Miot (École polytechnique)

Uniqueness for the Vlasov-Poisson system with unbounded density

We establish a uniqueness criterion for the Vlasov-Poisson system in two and three dimensions, allowing for densities that blow-up logarithmically. Moreover, we provide explicit examples of initial data for which the uniqueness condition is satisfied for all time.

Philip Morrison (Austin)

Sculpting Vlasov phase space

The conventional Cauchy problem (CP), where one considers initial data near a spectrally stable or unstable equilibrium state, has been widely studied in Vlasov theory by means of both analysis and computation. An alternative to this conventional approach is to begin with the dynamically accessible (DA) initial conditions proposed by the author and D. Pfirsch (1989). Such an initial condition is generated by applying external drive fields to an arbitrary phase space density for an interval of time, in order to prepare a state that serves as the initial data for subsequent undriven evolution. Examples of CP phenomena in the context of the Vlasov-Poisson (VP) system will be shown, including long-time "relaxation" to BGK-like states, with speculations about how to approach a rigorous justification of such limits. In addition, a variety of VP simulations with DA initial conditions will be presented. For example, complex non-sinusoidal oscillations obtained in the case of an abrupt turning off of the drive, and phase space sculpted by drives of multiple sinusoidal oscillations that open "resonances" will be presented. A taxonomy based on finite-dimensional Hamiltonian systems lore will be described for categorizing the rich set of states that can be generated for DA initial conditions.

Clément Mouhot (Cambridge)

Hölder continuity of solutions to Vlasov-Fokker-Planck type equations with rough coefficients

The celebrated De Giorgi-Nash theory about Hölder continuity of solutions to elliptic or parabolic equations with rough –i.e. merely measurable– coefficients in the late 1950s is a cornerstone of modern PDE analysis. We extend this theory to a class of kinetic equation of Vlasov-Fokker-Planck type ("hypoelliptic of type II" in the terminology of Hörmander) where a first-order hyperbolic operator interacts with a partially elliptic operator with rough coefficients. We also extend the theory of Moser about Harnack inequalities for these equations. This is a joint work with F. Golse, C. Imbert and A. Vasseur.

Toan Nguyen (Penn State)

Stability of a hot plasma in a solid torus

A collisionless plasma modeled by the relativistic Vlasov-Maxwell systems is considered in a solid torus, whose surface is assumed to be perfectly conducting and to reflect the particles specularly. I'll present criteria for the stability of equilibria under the assumption that the particle distributions and the electromagnetic fields depend only on the cross-sectional variables of the torus. This is a joint work with Walter Strauss.

Gerhard Rein (Bayreuth)

Gravitational collapse and the Vlasov equation

A self-gravitating homogeneous ball of a fluid with pressure zero where the fluid particles are initially at rest collapses to a point in finite time. We consider the problem of approximating this gravitational collapse by suitable solutions of the Vlasov-Poisson system. We also comment on the analogous problem in the general relativistic context.

Alexander Schekochihin (Oxford)

Phase mixing vs. nonlinear advection in drift-kinetic plasma turbulence

I will describe the simplest model kinetic system that is representative of the physical situation in magnetised plasmas in tokamaks (and, to some extent, conceptually, in space) — the near-Maxwellian drift kinetic plasma with Boltzmann electrons — and explain what kind of questions preoccupy plasma physicists and modellers interested in the turbulence of such a plasma. I will argue that the question centrally important to the understanding of kinetic turbulence is how the nonlinear advection of the perturbed distribution function by the fluctuating ExB flows competes with the phase mixing caused by particle streaming (which in a linear problem would lead to Landau damping). I will then outline a simple phenomenological scaling theory, in the spirit of Kolmogorov's theory of fluid turbulence, which focuses on the flows of free energy (perturbed entropy) in such a plasma. I will argue that very little free energy leaks into high velocity moments of the distribution, rendering the turbulent cascade in the energetically relevant part of the wave-number space essentially fluid-like. The free-energy content of the phase space does not diverge at infinitesimal collisionality (while it does for a linear problem); collisional heating due to long-wavelength perturbations vanishes in this limit (also in contrast with the linear problem, in which it occurs at the finite rate equal to the Landau-damping rate). The ability of the free energy to stay in the low velocity moments of the distribution is facilitated by the "anti-phase-mixing" effect, whose presence in the nonlinear system is due to the stochastic version of the plasma echo (the advecting velocity couples the phase-mixing and anti-phase-mixing perturbations). The partitioning of the wave-number space between the (energetically dominant) region where this is the case and the region where linear phase mixing wins is governed by the "critical balance" between linear and nonlinear timescales. Details can be found on arXiv:1508.05988.

Jacques Smulevici (Paris-Orsay)

A vector field method for kinetic transport equations with applications to classical and relativistic systems

The aim of our work is to develop a new approach to the study of kinetic transport equations by adapting the vector field method of Klainerman, a classical method traditionally used to obtain robust decay estimates for solutions of wave equations. I will start by explaining how this method can be used to prove decay estimates of velocity averages of solutions to classical and relativistic transport equations. In a second part, I will present several applications to the study of small data solutions of the Vlasov-Poisson and the massless or massive Vlasov-Norström systems. Joint work with David Fajman and Jérémie Joudioux.

Walter Strauss (Brown)

A body moving in a kinetic sea

Boundary interactions in kinetic theory are not well understood and are still in the early stages of mathematical analysis. In joint work with Xuwen Chen, we consider a free boundary problem with a diffusive boundary condition and with particles that can be subject to an external force. Our work is motivated by groundbreaking work of Caprino, Aoki, et al. We take a scenario that is largely one-dimensional and where the body's motion is initially close to an equilibrium velocity V. Does the body's velocity approach V? And, if it does, can we predict how it makes the approach? For instance, does it approach monotonically or not? How does the motion depend on the boundary condition and on the external force?

Martin Taylor (Cambridge)

Stability of Minkowski space for the massless Einstein–Vlasov system

Massless collisionless matter is described in general relativity by the massless Einstein-Vlasov system. Given asymptotically flat Cauchy data for this system which is sufficiently close, in a suitable sense, to the trivial solution, Minkowski space, the resulting maximal development exists globally in time and asymptotically decays appropriately. I will present a proof of this fact which can be shown via a reduction to the corresponding result for the vacuum Einstein equations, a monumental result first obtained by Christodoulou-Klainerman in the early 90s. A key step in the proof is to estimate certain Jacobi fields on the mass shell, a submanifold of the tangent bundle of the spacetime endowed with the Sasaki metric.

Cesare Tronci (Surrey)

Hybrid kinetic-fluid models for magnetized plasmas

Many physical contexts involve the interplay of different phenomena at different scales. The corresponding description requires the use of multi-physics models, whose mathematical formulation poses several challenges. Examples are found in the classical-quantum coupling in molecular dynamics or in the coupling between mean flow and fluctuation kinetics in turbulence. In plasma physics, the interaction of energetic particles (obeying kinetic theory) with a fluid bulk (obeying magnetohydrodynamics) requires formulating hybrid kinetic-fluid models, which are the subject of this talk. These models are often obtained by making assumptions on the equations of motion, although this operation may destroy fundamental properties such as energy balance. The use of symmetry techniques in mechanical systems is shown to provide a unifying framework for coupling nonlinear kinetic and fluid theories in a consistent way, thereby leading to new hybrid plasma models. Then, a comparison study is presented in terms of linear and nonlinear stability.