Dispersive Equations

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Preface

These notes were written to accompany a course on dispersive equations taught jointly by J. Ben-Artzi and A. Shao during the Autumn term of 2015 at the Taught Course Centre, to PhD students at the universities of Bath, Bristol, Oxford and Warwick as well as Imperial College London.

The general topic of *Dispersive Equations* is meant to represent our two research interests, *Kinetic Theory* (J. Ben-Artzi) and *Nonlinear Wave Equations* (A. Shao). While the latter is a classic "dispersive" topic, we include the former here as well due to the dispersive nature of the Vlasov equation, which is a transport equation in phase space.

This course is 16 hours in total which leaves merely 8 hours for each topic, including introduction. The introduction includes a crash course on basic methods in ordinary and partial differential equations, including the Cauchy problem, existence and uniqueness of solutions, the method of characteristics, Picard iteration, the Fourier transform and Sobolev spaces.

These notes are by no means meant to be complete and should only be treated as an assertional reference. Please let us know if you find any typos or mistakes. The main books we used when preparing the course were:

- Introductory materials
  - L. C. Evans, *Partial Differential Equations (second edition)*, AMS, 2010
  - T. Tao, *Nonlinear Dispersive Equations: Local and Global Analysis*, CBMS-AMS, 2006
  - H. Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer, 2011
- Kinetic Theory
  - F. Golse, *Lecture Notes (École polytechnique)*, `www.math.polytechnique.fr/~golse/M2/PolyKinetic.pdf`
  - R. T. Glassey, *The Cauchy Problem in Kinetic Theory*, SIAM, 1996
  - C. Mouhot, *Lecture Notes for Kinetic Theory Course (Cambridge)*, `https://cmouhot.wordpress.com/`

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- G. Rein, *Collisionless Kinetic Equations from Astrophysics – The Vlasov-Poisson System*, in Handbook of Differential Equations: Evolutionary Equations Volume 3, 2011

- Nonlinear Wave Equations
  - S. Selberg, *Lecture Notes for Nonlinear Wave Equations (Johns Hopkins)*, `http://www.math.ntnu.no/~sselberg/`
  - C. Sogge, *Lectures on Nonlinear Wave Equations*, International Press, 2006
  - L. Hörmander, *Lectures on Nonlinear Hyperbolic Differential Equations*, Springer-Verlag, 1997

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