MA 18 MIDTERM PRACTICE EXAM

Collaboration or use of online help is *forbidden* on the whole practice test until Tuesday at noon. After that you can discuss the problems with your peers, TAs and instructors.

You can turn in the five problems marked with $\star$ for *extra credit*. You should turn them in to your TA in recitations on Tuesday, October 12 at noon. If you can't make it to the recitations slide them under the door of Kassar 310 before Tuesday at noon. Other than the due time there is no time limit on how many hours you can spend on these problems. Write solutions nicely, explain your reasoning and make sure your handwriting is readable.

Completely correct solutions on these five problems can bring you extra 10 points on you midterm. The midterm will be worth 100 points.

(1) For each of the following questions indicate whether it is *true* or *false*. If it is true, *justify*. If it is false – *give a counter example*.
  (a) If $f(x,y,z) = 5x^3 + 2xy + 3z^2$, then $\nabla f(0,0,1)$ is perpendicular to the surface $f = 3$ at the point $(0,0,1)$.
  (b) The tangent plane to the surface $z = x^3 + 2y^2$ at the point $(1,1,3)$ is given by $z - 3 = 3(x-1) + 4(y-1)$.
  (c) Function $f(x,y) = \sqrt[3]{xy}$ is differentiable at every point.

(2)$\star$ Let $f : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by

$$f(x,y,z) = (e^{-2xy}, x^2 - z^2 - 4x + \sin(x+y+z))$$

and let $g : \mathbb{R}^2 \to \mathbb{R}$ be a function such that $g(1,0) = -1$ and $\nabla g(1,0) = \mathbf{i} - 3\mathbf{j}$.
  (a) Calculate the gradient of $g \circ f$ at the point $(0,0,0)$.
  (b) Find the equation of the tangent plane to the level set $g \circ f = -1$ at the point $(0,0,0)$.

(3)$\star$ Let $f(x,y,z) = x^2 + xy + y^2$.
  (a) Find the maximum and the minimum values of f along the circle $x^2 + y^2 = 1$.
  (b) Moving counterclockwise along the circle, is the function increasing or decreasing at the point $(1,0)$?

(4) Find the absolute maximum and minimum values of the function $f(x,y) = 5x^2 - 2y^2 + 10$ on the disk $x^2 + y^2 \le 1$, and specify where they are attained.

(5) Find the maximum rate of change of the function $f(x,y,z) = x^2 + 4y^2 + 9z^2$ at the point $P(2,-1,0)$ and the direction in which it occurs. Find the equation of the tangent plane to the ellipsoid $x^2 + 4y^2 + 9z^2 = 1$ at the point $P(\frac{1}{2}, \frac{1}{3}, -\frac{\sqrt{11}}{18})$.

(6)$\star$ Let $f(x,y,z) = \ln(z - \sqrt{3x^2 + 3y^2})$.
  (a) Find and sketch the natural domain of f.
  (b) Sketch and label level surfaces $f = k$ for $k < 0$, $k = 0$ and $k > 0$.

(7) Let $f = f(x,y)$. Express the gradient of f in terms of $\partial f/\partial r$ and $\partial f/\partial \theta$ where r and θ are polar coordinates.

(8)$\star$ Consider the curve in $\mathbb{R}^3$ defined by $\mathbf{r}(t) = (-t, t\sin \pi t, t\cos \pi t)$, for $t \in \mathbb{R}$.
  (a) Show that the graph of $\mathbf{r}(t)$ lies on the surface of a cone.
  (b) Sketch the curve.
  (c) Find a tangent line to the curve at $t = 1$.

(9)$\star$ Determine if the limit exists and if it exists find its value:

$$\lim_{(x,y)\to(0,0)} \frac{\cos(x+y) - 1}{(x+y)^2}.$$

(10) Let

$$f(x,y,z) = \frac{2x + 3y}{y + z}.$$

  (a) Find the local linear approximation of f at $A = (-1,1,1)$.
  (b) Compare the error of the approximation at $B = (-0.99, 0.99, 1.01)$ with the distance between A and B.