

MA3016 MODULE SUMMARY

(1) Main theorems and proofs

- (a) The Wave Equation
 - (i) Solutions have the form $f(x + ct) + g(x - ct)$.
 - (ii) d'Alembert's formula.
- (b) The Diffusion Equation.
 - (i) Properties of the Gaussian.
 - (ii) Weak Maximum Principle.
 - (iii) Statement of the Strong Maximum Principle (no proof).
 - (iv) Uniqueness of Solutions.
 - (v) Stability of Solutions.
- (c) Fourier Series
 - (i) Orthogonality of sines/cosines
 - (ii) Green's Second Identity.
 - (iii) Theorems regarding eigenvalues/eigenfunctions of $\mathcal{L}_D, \mathcal{L}_N, \mathcal{L}_P$.
 - (iv) Convergence of Fourier Series.
- (d) Laplace's Equation.
 - (i) Uniqueness of Solutions (relies on max. principle, below)
 - (ii) Poisson's Formula.
 - (iii) Mean Value Property.
 - (iv) Strong Maximum Principle.

(2) Techniques and other properties

- (a) **First order PDEs**
- (b) The "energy method".
- (c) The backwards diffusion equation is ill-posed.
- (d) Coefficients of a Fourier series.
- (e) Notions of Convergence
- (f) The Laplacian in polar coordinates.
- (g) **Separation of Variable**
 - (i) Harmonics
 - (ii) Frequencies
 - (iii) For Laplace: separate variables in rectangles, polar coordinates.